Diploma Programme

# Mathematics mind map

## Applications and interpretation SL

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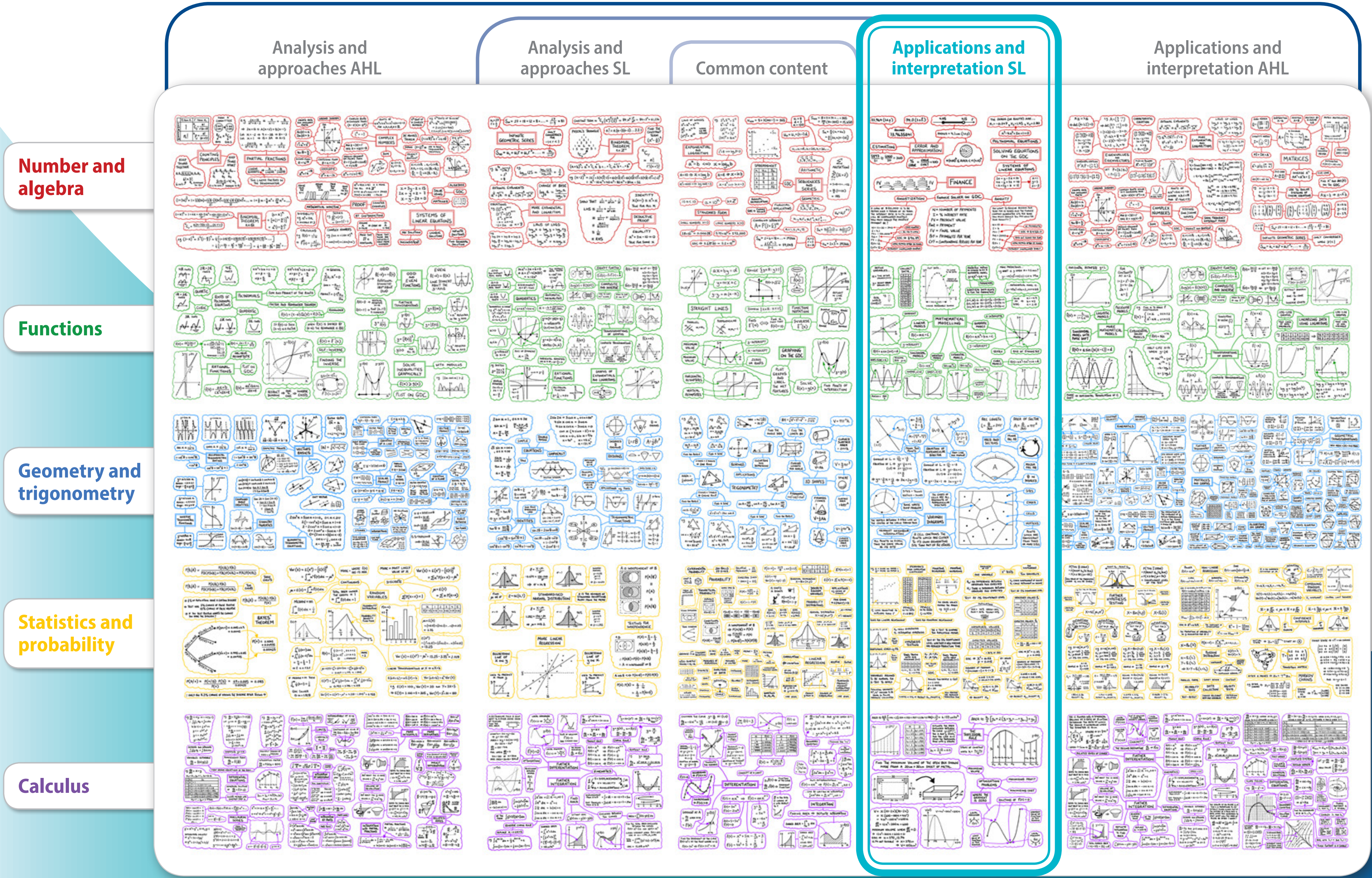
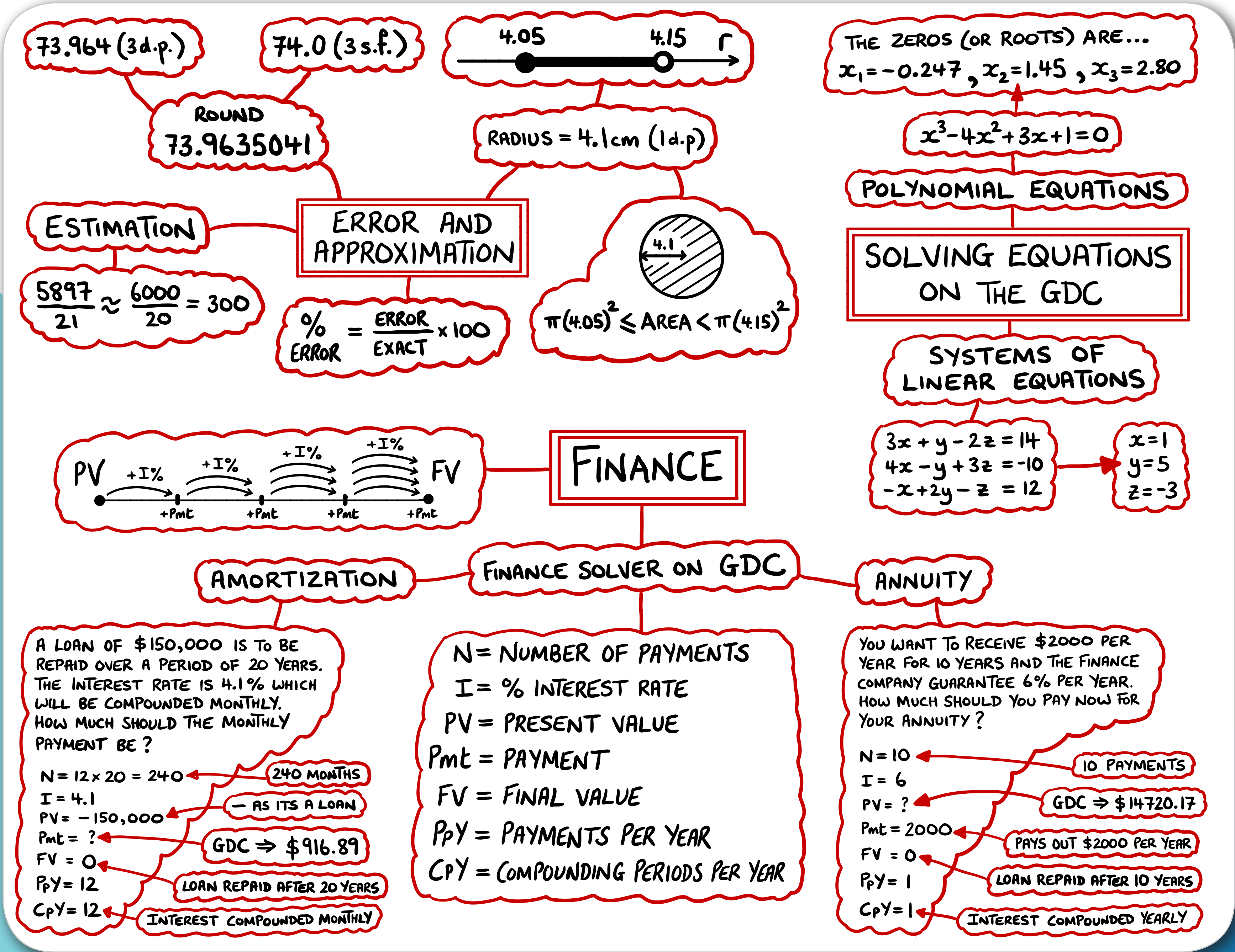
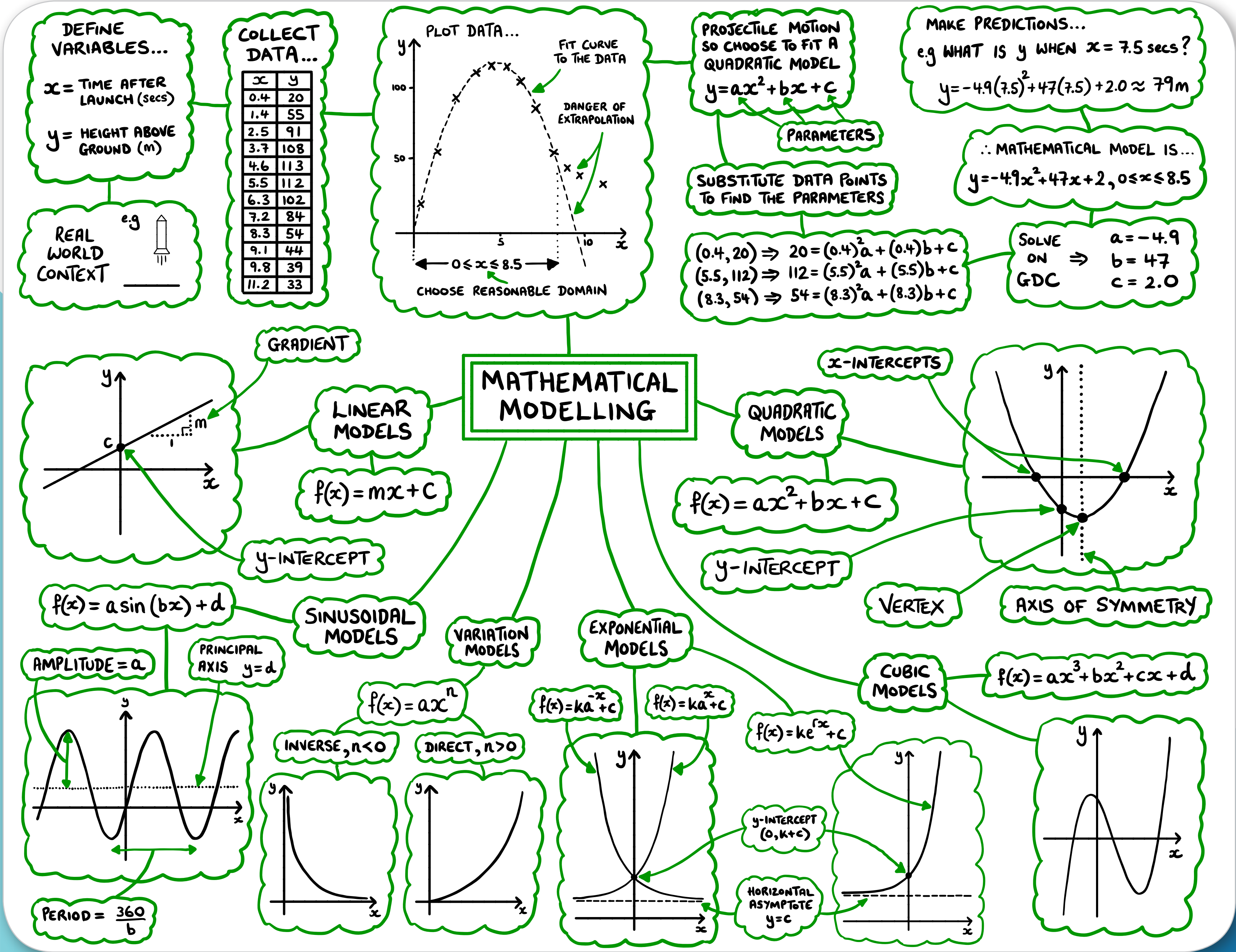


# Number and algebra



## Applications and interpretation SL

# Functions

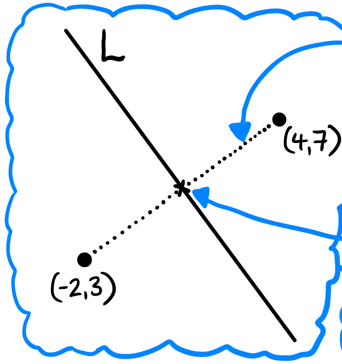


# Geometry and trigonometry



## Equation of perpendicular bisector

### From two points




$m = \frac{7-3}{4--2} = \frac{4}{6} = \frac{2}{3}$

$M = \left(\frac{-2+4}{2}, \frac{3+7}{2}\right) = (1,5)$

Gradient of L $= \frac{-1}{2/3} = \frac{-3}{2}$

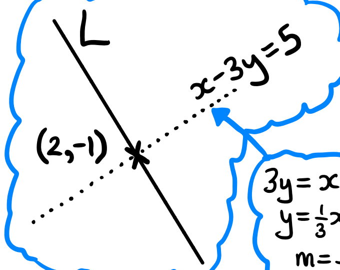
Equation of L $\Rightarrow y = -\frac{3}{2}x + c$

$(1,5) \Rightarrow 5 = -\frac{3}{2}(1) + c$

$\therefore c = \frac{13}{2}$

$\Rightarrow y = -\frac{3}{2}x + \frac{13}{2} \Rightarrow 3x + 2y = 13$

### From line and midpoint




$3y = x - 5$

$y = \frac{1}{3}x - \frac{5}{3}$

$m = \frac{1}{3}$

Gradient of L $= \frac{-1}{1/3} = -3$

Equation of L $\Rightarrow y = -3x + c$

$(2,-1) \Rightarrow 2 = -3(-1) + c$

$\therefore c = -1$

$\Rightarrow y = -3x - 1 \Rightarrow 3x + y = -1$

## Arcs and sectors

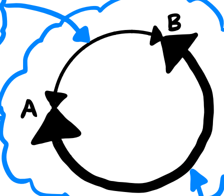
Arc length

$l = \frac{\theta}{360} \times 2\pi r$

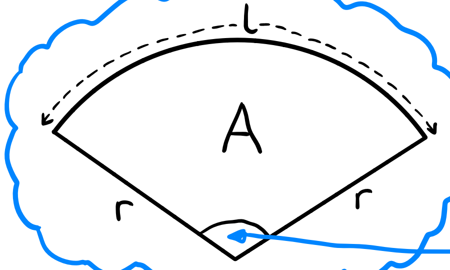
Area of sector

$A = \frac{\theta}{360} \times \pi r^2$

Minor arc AB




Major arc AB



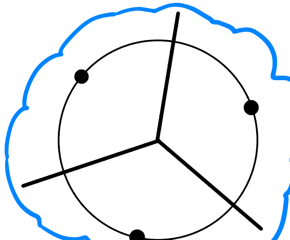

$\theta$ in degrees

## Voronoi diagrams

Toxic waste dump problem

Sites = Towns

Vertices = Dumps



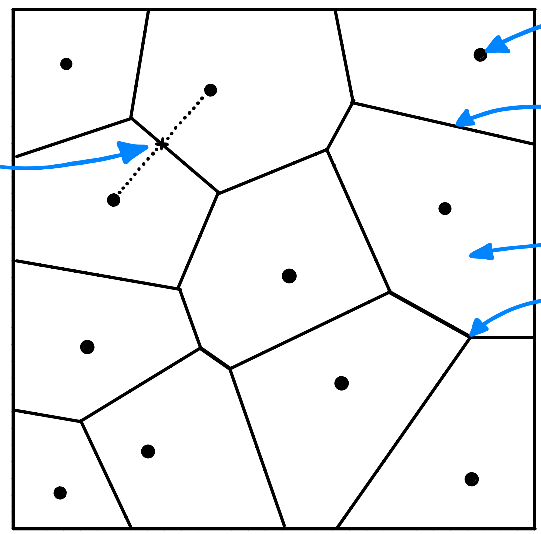
The vertex between 3 sites lies at the centre of the circle through them.

Nearest neighbour interpolation

All points in a cell take the same value as its site

The edges of cells are all segments of perpendicular bisectors

A cell contains the points which are closer to it's own generating site than any of the others.



Sites

Edges

Cells

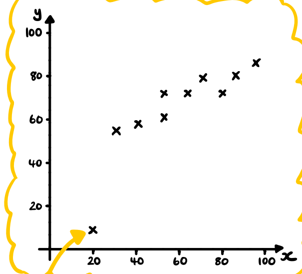
Vertices

Points which are equidistant from three or more sites

# Statistics and probability



e.g $x$ = SCORE ON PAPER 1, $y$ = SCORE ON PAPER 2



$r_s$ LESS SENSITIVE TO OUTLIERS THAN $r$

SPEARMAN'S RANK CORRELATION COEFFICIENT

| $x$ | $y$ |
|---|---|
| 20 | 9 |
| 30 | 56 |
| 41 | 58 |
| 53 | 61 |
| 53 | 72 |
| 64 | 72 |
| 71 | 79 |
| 80 | 72 |
| 85 | 80 |
| 96 | 85 |

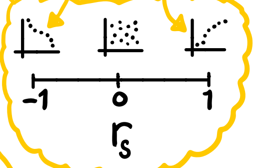
| rank$_x$ | rank$_y$ | d |
|---|---|---|
| 1 | 1 | 0 |
| 2 | 2 | 0 |
| 3 | 3 | 0 |
| 4.5 | 4 | 0.5 |
| 4.5 | 6 | -1.5 |
| 6 | 6 | 0 |
| 7 | 8 | -1 |
| 8 | 6 | 2 |
| 9 | 9 | 0 |
| 10 | 10 | 0 |

PPMCC ON THE DATA $r = 0.842$

PPMCC ON THE RANKS $r_s = 0.954$

MONOTONE DECREASING

MONOTONE INCREASING



FOR EQUAL VALUES AVERAGE THE RANKS e.g $\frac{4+5}{2} = 4.5$

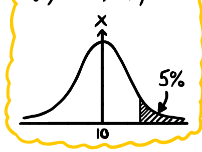
TESTS FOR LINEAR RELATIONSHIP

TESTS FOR MONOTONIC RELATIONSHIP

HYPOTHESIS TESTING

$H_0$: NULL HYPOTHESIS
$H_1$: ALTERNATIVE HYPOTHESIS

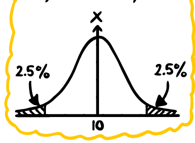
$H_0: \mu = 10$, $H_1: \mu > 10$



ONE-TAILED TEST

SIGNIFICANCE LEVELS e.g 5%

$H_0: \mu = 10$, $H_1: \mu \neq 10$



TWO-TAILED TEST

THE t-TEST TO COMPARE TWO POPULATION MEANS

TEST AT THE 5% SIGNIFICANCE LEVEL WHETHER A NEW MACHINE HAS REDUCED PRODUCTION TIME.

TIMES TAKEN WITH OLD MACHINE (secs)

| | | | | | |
|---|---|---|---|---|---|
| 51.9 | 52.7 | 52.6 | 52.5 | 53.1 | 54.0 |
| 53.6 | 53.0 | 53.3 | 52.9 | 51.7 | 54.1 |

$n_1 = 12$, $\bar{x}_1 = 52.95$, $s_1 = 0.7416$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 > \mu_2$

ONE-TAILED TEST

TIMES TAKEN WITH NEW MACHINE (secs)

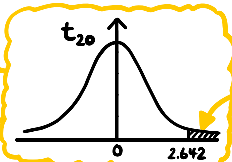
| | | | | |
|---|---|---|---|---|
| 52.1 | 51.3 | 52.4 | 53.2 | 51.8 |
| 51.0 | 51.8 | 52.8 | 52.3 | 52.7 |

$n_2 = 10$, $\bar{x}_2 = 52.14$, $s_2 = 0.6835$

GDC

VARIABLES ASSUMED TO BE NORMAL FOR t-TEST TO APPLY

POPULATION VARIANCES $\sigma_1^2$, $\sigma_2^2$ ASSUMED TO BE EQUAL SO POOLED VARIANCE CALCULATED



POOLED TWO SAMPLE t-TEST
$t = 2.642$
$p = 0.00782$
$df = 20$

$0.00782 < 5\% \Rightarrow$ REJECT $H_0$, ACCEPT $H_1$

NEW MACHINE HAS REDUCED PRODUCTION TIME

## $\chi^2$ TESTS

GOODNESS OF FIT

ONE VARIABLE

$H_0$: NO DIFFERENCE BETWEEN OBSERVED AND EXPECTED
$H_1$: THERE IS DIFFERENCE BETWEEN OBSERVED AND EXPECTED

TEST AT 1% SIGNIFICANCE LEVEL

POPULATION
RED = 40% GREEN = 20%
BLUE = 30% YELLOW = 10%
SAMPLE

OBSERVED VALUES

| COLOUR | RED | BLUE | GREEN | YELLOW | TOTAL |
|---|---|---|---|---|---|
| $f_o$ | 74 | 40 | 17 | 19 | 150 |

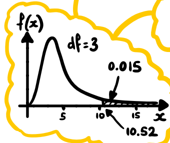
EXPECTED VALUES

| COLOUR | RED | BLUE | GREEN | YELLOW | TOTAL |
|---|---|---|---|---|---|
| $f_E$ | 60 | 45 | 30 | 15 | 150 |

= 30% OF 150

GDC $\Rightarrow \chi^2 = 10.52$, $p = 0.015$

DEGREES OF FREEDOM = (COLUMNS − 1) = 4 − 1 = 3



$0.015 > 0.01$
p-VALUE > SIG. LEVEL

$10.52 < 11.34$
$\chi^2_{calc} < \chi^2_{crit}$

$\Rightarrow$ ACCEPT $H_0$

BY HAND $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

FOR INDEPENDENCE

TWO VARIABLES

$H_0$: CHOICE INDEPENDENT OF INCOME
$H_1$: CHOICE DEPENDENT ON INCOME

TEST AT 5% SIGNIFICANCE LEVEL

OBSERVED VALUES $f_o$

| | A | B | C | D | |
|---|---|---|---|---|---|
| LOW | 13 | 19 | 7 | 11 | 50 |
| MEDIUM | 5 | 9 | 5 | 11 | 30 |
| HIGH | 12 | 12 | 13 | 3 | 40 |
| | 30 | 40 | 25 | 25 | 120 |

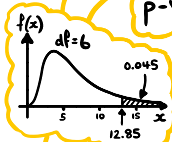
EXPECTED VALUES $f_E$

| | A | B | C | D | |
|---|---|---|---|---|---|
| LOW | 12.5 | 16.7 | 10.4 | 10.4 | 50 |
| MEDIUM | 7.5 | 10.0 | 6.3 | 6.3 | 30 |
| HIGH | 10.0 | 13.3 | 8.3 | 8.3 | 40 |
| | 30 | 40 | 25 | 25 | 120 |

$= \frac{40}{120} \times \frac{30}{120} \times 120 = 10$

GDC $\Rightarrow \chi^2 = 12.85$, $p = 0.045$

DEGREES OF FREEDOM
= (ROWS − 1)(COLUMNS − 1)
= (3 − 1)(4 − 1) = 2 × 3 = 6



$0.045 < 0.05$
p-VALUE < SIG. LEVEL

$12.85 > 12.59$
$\chi^2_{calc} > \chi^2_{crit}$

$\Rightarrow$ REJECT $H_0$, ACCEPT $H_1$

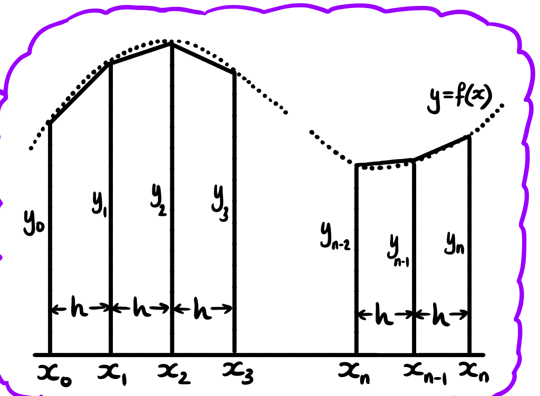
# Calculus



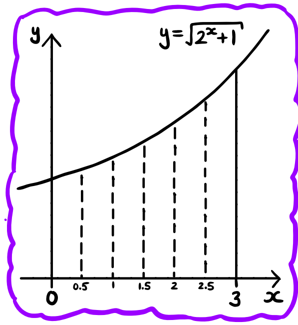
## The Trapezoidal Rule

$$\text{AREA} \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n\right]$$

STRIPS OF CONSTANT WIDTH $h = \frac{x_n - x_0}{n}$




ESTIMATE THE AREA ENCLOSED BETWEEN THE CURVE $y=\sqrt{2^x+1}$, THE $x$ AND $y$ AXES, AND THE LINE $x=3$ USING THE TRAPEZOIDAL RULE WITH 6 STRIPS.




| $x$ | $y$ |
|---|---|
| $x_0 = 0$ | $y_0 = 1.414$ |
| $x_1 = 0.5$ | $y_1 = 1.554$ |
| $x_2 = 1$ | $y_2 = 1.732$ |
| $x_3 = 1.5$ | $y_3 = 1.957$ |
| $x_4 = 2$ | $y_4 = 2.236$ |
| $x_5 = 2.5$ | $y_5 = 2.580$ |
| $x_6 = 3$ | $y_6 = 3.000$ |

$$h = \frac{3-0}{6} = 0.5$$

$$\text{AREA} \approx \frac{0.5}{2}\left[1.414 + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580) + 3\right] = 6.133 \text{ units}^2$$

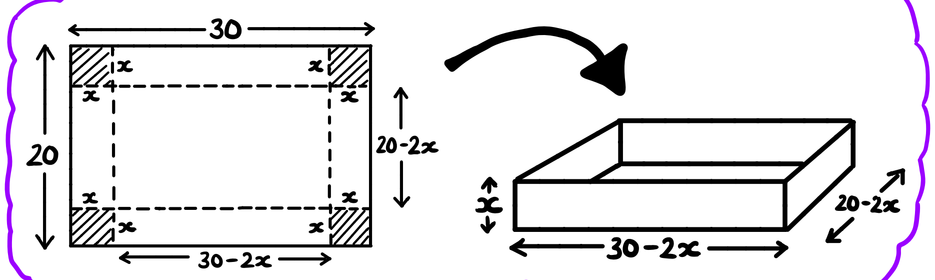
## Optimisation Problems

MAXIMISING VOLUME

MAXIMISING PROFIT

MINIMISING COST

FIND THE MAXIMUM VOLUME OF THE OPEN BOX SHOWN MADE FROM A 20cm × 30cm SHEET OF METAL.




$V = x(20-2x)(30-2x)$

$= x(600 - 100x + 4x^2)$

$= 4x^3 - 100x^2 + 600x$

$\frac{dV}{dx} = 12x^2 - 200x + 600$

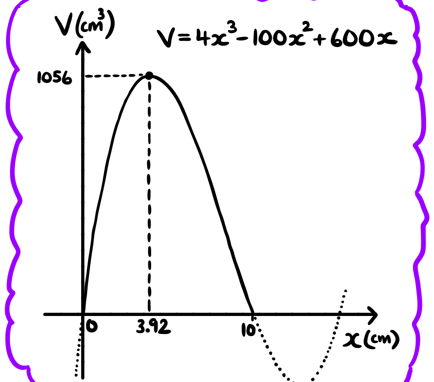
MAXIMUM VOLUME WHEN $\frac{dV}{dx} = 0$

$\Rightarrow 12x^2 - 200x + 600 = 0$

GDC $\Rightarrow x = 3.92, 12.74$

12.74 NOT POSSIBLE $\Rightarrow x = 3.92$cm

$\Rightarrow V = 1056\text{cm}^3$




WHERE THE GRADIENT IS ZERO

SOLUTIONS OF $f'(x) = 0$